3.a $\quad E[X] = \mu + \frac{\sigma}{\sqrt{2\pi}} \int_{-\infty}^{\infty} z e^{-\frac{z^2}{2}} dz$

La $g(z)$ oppfylle $g(-z) = -g(z)$

Då får vi $\quad \int_{-\infty}^{\infty} g(z)\,dz = \int_{-\infty}^{0} g(z)\,dz + \int_{0}^{\infty} g(z)\,dz$

$$= -\int_{-\infty}^{0} g(-z)\,dz + \int_{0}^{\infty} g(z)\,dz = -\int_{0}^{\infty} g(z)\,dz + \int_{0}^{\infty} g(z)\,dz = 0$$

$\| (u = -z)$

$$\int_{\infty}^{0} g(u)\,du$$

ɔ: $E[X] = 0$.

$$Var[X] = E[(X-\mu)^2] = \frac{1}{\sqrt{2\pi}\,\sigma} \int_{-\infty}^{\infty} (x-\mu)^2 e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

$z = \frac{x-\mu}{\sigma}$

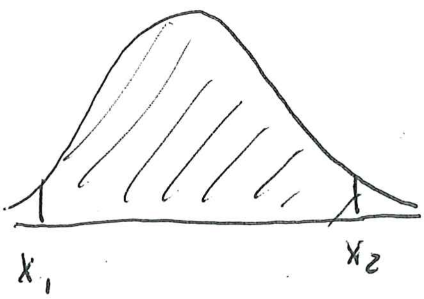
$$= \frac{1}{\sqrt{2\pi}\,\sigma} \int_{-\infty}^{\infty} \sigma^2 z^2 e^{-\frac{z^2}{2}} \sigma\,dz = \frac{\sigma^2}{\sqrt{2\pi}} \int_{-\infty}^{\infty} z^2 e^{-\frac{z^2}{2}} dz$$

$$= \frac{\sigma^2}{\sqrt{2\pi}} \left(\left[-z e^{-\frac{z^2}{2}} \right]_{-\infty}^{\infty} + \int_{-\infty}^{\infty} e^{-\frac{z^2}{2}} dz \right) = \sigma^2 \left[0 + \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{z^2}{2}} dz \right] = \sigma^2.$$

$X \sim N(\mu, \sigma^2)$. $\quad P(x_1 \le X \le x_2) = \frac{1}{\sqrt{2\pi}\,\sigma} \int_{x_1}^{x_2} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$

$z = \frac{x-\mu}{\sigma}$

$$= \frac{1}{\sqrt{2\pi}} \int_{z_1}^{z_2} e^{-\frac{z^2}{2}} dz = P(z_1 \le Z \le z_2)$$



der $z_2 = \frac{x_2-\mu}{\sigma}$ og $z_1 = \frac{x_1-\mu}{\sigma}$

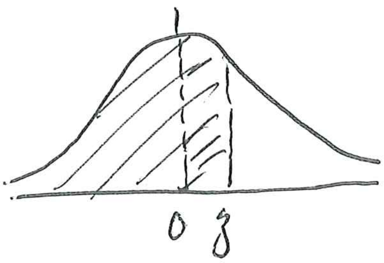
$ɔ: \quad P(x_1 \le X \le x_2) = \int_{z_1}^{z_2} f(x, 0, 1)\, dz$

<u>Definisjon 6.1</u> Ein normalfordelt variabel med forventing 0 og varians 1 seiast å vere standard normalfordelt.

<u>Utrekning av sannsyn i normalfordelinga</u>

Tabellen gjev:

$P(Z \le z)$ ɔ:



der $Z \sim N(0,1)$

<u>Eks.</u> $X \sim N(50, 10^2)$

$P(X \le 62) = P\left(\frac{X-50}{10} \le \frac{62-50}{10}\right) = P(Z \le 1.2) = 0.8849$

$P(X \le 45) = P\left(\frac{X-50}{10} \le \frac{45-50}{10}\right) = P(Z \le -0.5) = 0.3085$

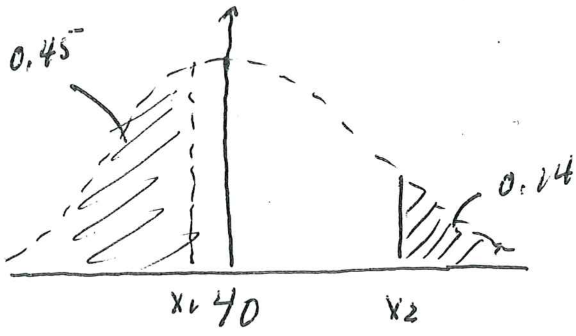
$P(45 \le X \le 62) = P\left(\frac{45-50}{10} \le \frac{X-50}{10} \le \frac{62-50}{10}\right) = P(Z \le 1.2) - P(Z \le -0.5$

$= 0,8849 - 0.3085 = 0.5764$

<u>Eks.</u> $X \sim N(40, 6^2)$

Finn x_1 og x_2 s.a.

$P(X \le x_1) = 0.45, \quad P(X \ge x_2) = 0.14$



$$P(X \le x_1) = P\left(\frac{X-40}{6} \le \frac{x_1-40}{6}\right) = P\left(Z \le \frac{x_1-40}{6}\right) = 0{,}45$$

$\partial;$ $$\frac{x_1-40}{6} = -0{,}13 \Rightarrow x_1 = 40-0{,}13\cdot 6 = 39{,}22$$

$$P(X \le x_2) = P\left(\frac{X-40}{6} \le \frac{x_2-40}{6}\right) = P\left(Z \le \frac{x_2-40}{6}\right) = 0{,}86$$

$\partial;$ $$\frac{x_2-40}{6} = 1{,}08 \Rightarrow x_2 = 40+1{,}08\cdot 6 = 46{,}48.$$

6.5 Normaltilnærming til binomisk fordeling

Flyselskap: Flyrute med plass til 168 passasjerar

Har selt ut 178 billettar

90% av dei som har kjøpt billett møter.

A = Ein som har kjøpt billett møter

La X = talet på passasjerar som møter

$X \sim B(178, 0.9)$

Skal finne $P(\text{ikkje alle får plass}) = P(169 \leq X \leq 178)$

$$X = \sum_{i=1}^{178} X_i, \quad \text{der} \quad X_i = \begin{cases} 1 & \text{dersom passasjer } i \text{ møter}, \quad i = 1, 2, \dots, 178 \\ 0 & \text{elles.} \end{cases}$$

Rimeleg at X er tilnærma normalfordelt.

$E[X] = np = 178 \cdot 0.9 = 160.2$

$Var[X] = np(1-p) = 178 \cdot 0.9 \cdot 0.1 = 16.02$

$$P(169 \leq X \leq 178) \approx P\left(\frac{169 - 160.2}{\sqrt{16.02}} \leq \frac{X - 160.2}{\sqrt{16.02}} \leq \frac{178 - 160.2}{\sqrt{16.02}}\right)$$

$$= P(2.198 \leq Z \leq 4.45) = \Phi(4.45) - \Phi(2.198)$$

$$= 1 - 0.986 = 0.014$$

Alternativ kan ein bruke kontinuitetsapproksimasjon (ofte betre)

$$P(169 \leq X \leq 178) \approx P\left(\frac{168.5 - 160.2}{\sqrt{16.02}} \leq \frac{X - 160.2}{\sqrt{16.02}} \leq \frac{178.5 - 160.2}{\sqrt{16.02}}\right)$$

$$= P(2.07 \leq Z \leq 4.57) = 1 - 0.981 = 0.019$$

Fungerer greit for $\left.\begin{matrix} np \\ n(1-p) \end{matrix}\right\} \geq 5$

6.6. Gamma og eksponential fordeling

$X \sim$ Poisson prosess. $\quad P(X=x) = \dfrac{(\lambda t)^x e^{-\lambda t}}{x!}$

La T = tida til 1. hending

$F_T(t) = P(T \leq t) = 1 - P(T > t) = 1 - P(X = 0$ i intervallet $[0,t])$

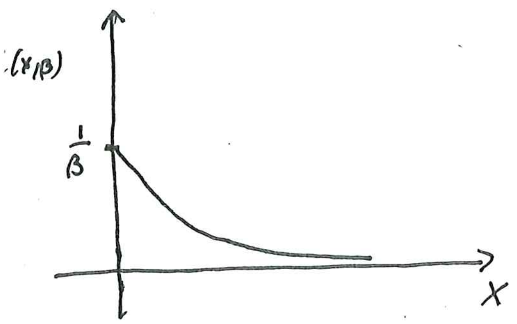
$= 1 - \dfrac{(\lambda t)^0 e^{-\lambda t}}{0!} = 1 - e^{-\lambda t}, \quad t \geq 0$

$$f_T(t) = \begin{cases} \lambda e^{-\lambda t}, & t > 0 \\ 0, & \text{elles.} \end{cases}$$

Ei fordeling med sannsynstettleik gitt ved

$$f(x, \beta) = \begin{cases} \frac{1}{\beta} e^{-\frac{x}{\beta}}, & x > 0 \\ 0, & \text{elles} \end{cases}$$

blir kalla ei eksponential fordeling



$$E[X] = \int_0^\infty x \cdot \frac{1}{\beta} e^{-\frac{x}{\beta}} dx = \left[-x e^{-\frac{x}{\beta}}\right]_0^\infty + \int_0^\infty e^{-\frac{x}{\beta}} dx$$

$$= \left[-\beta e^{-\frac{x}{\beta}}\right]_0^\infty = \beta$$

$$E[X^2] = \int_0^\infty x^2 \cdot \frac{1}{\beta} e^{-\frac{x}{\beta}} dx = \left[-x^2 e^{-\frac{x}{\beta}}\right]_0^\infty + 2\int_0^\infty x e^{-\frac{x}{\beta}} dx$$

$$= 2 \cdot \beta \int_0^\infty x \cdot \frac{1}{\beta} e^{-\frac{x}{\beta}} dx = 2\beta^2 \Rightarrow Var[X] = 2\beta^2 - \beta^2 = \beta^2$$

$$\Rightarrow SD(X) = \beta = E[X].$$